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Underexpanded Jets of Liquid-Gas Bubble Mixtures

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Nomenclature

D = nozzle diameter
L = cell length
P_a = ambient pressure
P_o = stagnation pressure
P^* = static pressure in throat at choking
δ = concentration (volume of gas to volume of liquid)
θ = jet angle

THE speed of a low-frequency pressure pulse traveling in a liquid suspension of gas bubbles is known to be quite low due to the combination of the high compressibility and density of the mixture.[1] For example, the speed of a pulse traveling in a 50% volumetric concentration of air bubbles in water at atmospheric pressure is about 65 fps. Because of this fact it is easy to produce compressible supersonic flows of these mixtures in the laboratory as was shown in an early experiment by Tangren, Dodge and Seifert.[1] A subsequent study by Campbell and Pitcher[2] demonstrated the existence of shock-like regions of rapid bubble compression. Miur and Eichorn[3] extended the experimental work of Tangren et al. to a two-dimensional convergent-divergent nozzle and in a recent work, Eddington[4] constructed and tested a continuous flow tunnel and examined flow over wedge bodies.

The purpose of this Note is to discuss the free expansion of a bubbly flow. In Ref. 1 it was observed that a water-air bubble mixture broke up into a spray when exhausting to atmospheric pressure at supersonic velocities. In a study reported here it was found that a continuous jet having a structure similar to an underexpanded gas jet exiting from a convergent nozzle is also possible under some conditions. This mode of jet structure has apparently not been observed previously for these mixtures.

A suspension of fine air bubbles in glycerine was formed by high-speed mechanical mixing. A small amount (6 g/liter) of sodium lauryl sulphate was added to the glycerine to obtain higher bubble concentrations. The density of the mixture was measured directly from the weight of a known volume. A freejet was produced in a blow-down type facility by accelerating the mixture through a $\frac{1}{4}$-in. nozzle from an upstream reservoir at atmospheric pressure into an evacuated chamber.

Tests were run at various pressure ratios for several gas concentrations. Typical results are shown in Fig. 1 for a volumetric ratio of gas to liquid $\delta = 0.45$. For this concentration of air in glycerin, calculations based on Ref. 1 indicate that the jet is choked for pressure ratios above $P^*/P_o = 0.455$. Choking was confirmed in independent measurements of flow rate and static pressure at the throat.

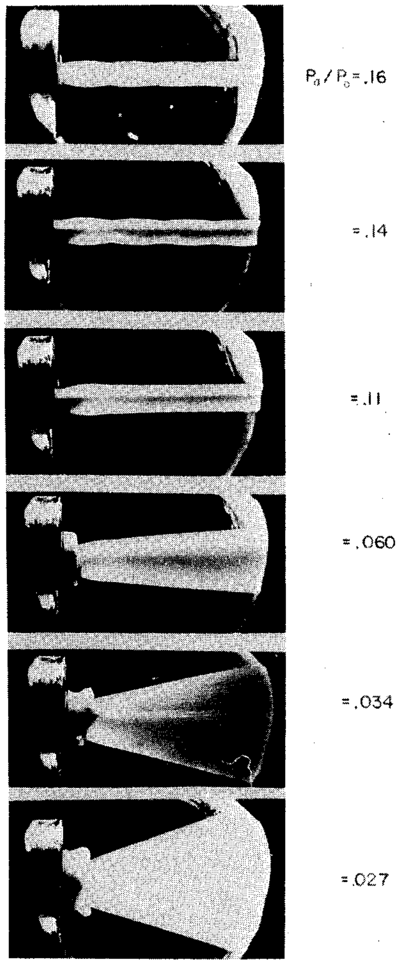

Fig. 1 Jet structure at various pressure ratios ($\delta_0 = 0.45$).

The photographs show that for pressure ratios above choking the jet develops a cell-like periodic structure which increases in size as the pressure ratio is increased. Figure 2 shows the increase in cell size at a relatively low gas concentration as a function of ambient pressure above critical ($P^* - P_a/P_o$). At higher concentration such as those shown in the photograph, the increase in cell size is considerably less. Qualitatively this behavior is analogous to the supersonic

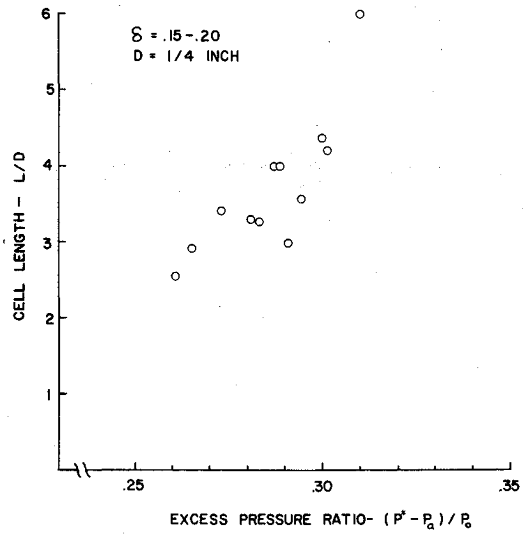

Fig. 2 Cell dimensions for continuous jet.

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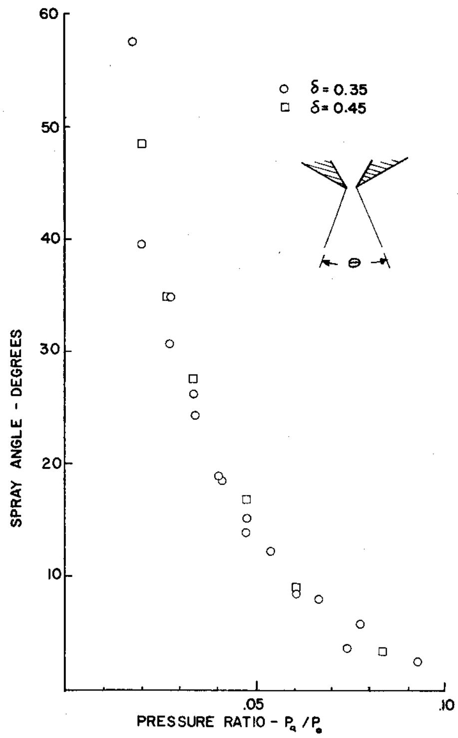

Fig. 3 Spray angle.

free expansion of a gas.[5] The jet exits at a pressure above ambient, expands to a pressure below ambient and contracts to recover the initial exit pressure. The process is then repeated giving the periodic jet boundary until the jet is dissipated.

As the pressure ratio across the jet is increased, the spray mode reported in Ref. 1 develops. The angle of the spray is quite sensitive to pressure ratio as shown in Fig. 3. Data for other nozzles and other concentrations show similar trends. At higher concentrations a secondary flow develops with the spray (see Fig. 1). This appears to be a thin fluid layer which separates at the edge of the nozzle and is sometimes entrained downstream in the spray.

Breakup occurred over a range of pressure ratios and was difficult to reproduce partially due to mechanical factors, i.e., nozzle configuration, vibration, etc. No detailed quantitative measurements of breakup were attempted. Tangren et al. have suggested that the jet is disrupted by the rapid expansion of the jet downstream of the nozzle. This explanation appears to be quite reasonable in view of the present experiment since breakup can be prevented by using a liquid of higher viscosity than water. Presumably the bubble expansion is less violent in the more viscous glycerine. We have further observed that some of the continuous jet structure remains even while the spray is developing. The jet structure for pressure ratios of $P_a/P_o = 0.11$ and 0.06 show this effect.

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A High-Performance Shock Tube with Air Driver

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To produce shock waves in a shock tube with Mach numbers of the order of 10 in air, it is usual to employ as driver gas either hydrogen, or a combustible mixture of oxygen and hydrogen diluted by helium. However, the use of air offers advantages. For example, in shock tunnel operation this would render the consequences of contamination of the test gas by driver gas less serious than they are in present high performance facilities,[1] while in shock tube operation it offers improved safety and economy.

The experiments reported here were associated with development of a shock tube for use in a university undergraduate laboratory. The purpose was to achieve shock speeds which would allow experiments involving substantial real gas effects, in a facility which would be both safe and economical when operated by undergraduate students. The facility is shown schematically in Fig. 1. Essentially, it is a double diaphragm or "buffered" free piston shock tube.[2,3] The free piston driver, shown in Fig. 1a, consists of a compression tube 1.98 m long and 80.5 mm in diameter, together with a piston driver reservoir 1.0 m long and of the same diameter. The primary diaphragm was located at the downstream end of the compression tube, and was followed by an intermediate shock tube 38 mm in diameter and 0.83 m long. The intermediate tube terminated at a flanged mount for the secondary diaphragm, and was followed by a test shock tube 38 mm in diameter and 2.75 m long. This opened into a dump tank at the downstream end.

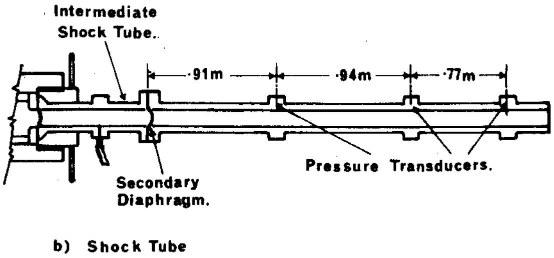

Fig. 1 Schematic general arrangement of shock tube facility.

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